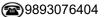


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ON GENERALIZED PSEUDO-PROJECTIVE CURVATURE TENSOR OF PARA-KENMOTSU MANIFOLDS

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Abstract

The object of the present paper is to generalize pseudo-projective curvature tensor of para-Kenmotsu manifold with the help of a new generalized (0,2) symmetric tensor $\mathcal Z$ introduced by Mantica and Suh. Various geometric properties of generalized pseudo-projective curvature tensor of para-Kenmotsu manifold have been studied. It is shown that a generalized pseudo-projectively ϕ -symmetric para-Kenmotsu manifold is an Einstein manifold.

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Key words: Pseudo-projective curvature tensor, para-Kenmotsu manifold, Einstein manifold, η -Einstein manifold, Generalized pseudo-projective curvature tensor.

1 Introduction

The projective tensor is one of the major curvature tensors. The study of pseudo-projective curvature tensor has been a very attractive field for investigations in the past decades. A tensor field \overline{P} was defined and studied in 2002 by Bhagwat Prasad [18] on a Riemannian manifold of dimension n, which includes projective curvature tensor P. This tensor field \overline{P} referred to as pseudo-projective curvature tensor. In 2011, H.G. Nagaraja and G. Somashekhara [14] extended pseudo-projective curvature tensor in Sasakian manifolds. After 2012, the pseudo-projective curvature tensor analysis in LP-Sasakian manifolds was resumed by

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Y.B. Maralabhavi and G.S. Shivaprasanna [12]. In 2016, S. Mallick, Y.J. Sub and U.C. De [11] defined and studied a space time with pseudo-projective curvature tensor. Subsequently, several researchers performed a study of pseudo-projective curvature tensor in a number of directions, such as [4, 5, 13, 15, 17, 21, 22]. The pseudo-projective curvature tensor is defined by [18]

$$\overline{P}(X,Y,U) = aR(X,Y,U) + b[S(Y,U)X - S(X,U)Y] - \frac{r}{n} \left(\frac{a}{n-1} + b\right) [g(Y,U)X - g(X,U)Y],$$
(1)

where a and b are constants such that a, b \neq 0 and R is the curvature tensor, S is the Ricci tensor and r is the scalar curvature tensor.

The notion of an almost para-contact manifold was introduced by I. Sato [19]. Since the publication of [26], paracontact metric manifolds have been studied by many authors in recent years. The importance of para-Kenmotsu geometry, have been pointed out especially in the last years by several papers highlighting the exchanges with the theory of para-Kähler manifolds and its role in semi-Riemannian geometry and mathematical physics [3, 7, 8, 20].

In this paper, we consider the generalized pseudo-projective curvature tensor of para-Kenmotsu manifolds and study some properties of generalized pseudo-projective curvature tensor. The organisation of the paper is as follows: After preliminaries on para-Kenmotsu manifold in Section 2, we describe briefly the generalized pseudo-projective curvature tensor on para-Kenmotsu manifold in Section 3 and also we study some properties of generalized pseudo-projective curvature tensor in para-Kenmotsu manifold. In Section 4, we study a generalized pseudo-projectively semi-symmetric para-Kenmotsu manifold is an η Einstein manifold. Further in the Section 5, we show that a generalized pseudo-projectively Ricci semi-symmetric para-Kenmotsu manifold is either Einstein manifold or $\psi = \frac{an(n-1)+ra+br(n-1)}{bn(n-1)}$ on it. In the last section we show that the generalized pseudo-projectively ϕ -symmetric para-Kenmotsu manifold is an Einstein manifold.

2 Preliminaries

An *n*-dimensional differentiable manifold M^n is said to have almost paracontact structure (ϕ, ξ, η) , where ϕ is a tensor field of type (1, 1). ξ is a vector field known as characteristic vector field and η is a 1-form satisfying the following relations

$$\phi^{2}(X) = X - \eta(X)\xi, \qquad (2)$$

$$\eta(\phi X) = 0, \tag{3}$$

 $\phi(\xi) = 0,$ (4)

and

 $\eta(\xi) = 1.$

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A differentiable manifold with almost para-contact structure (ϕ, ξ, η) is called an almost para-contact manifold. Further, if the manifold M^n has a semi-Riemannian metric g satisfying

$$\eta(X) = q(X, \xi) \tag{6}$$

and

$$g(\phi X, \phi Y) = -g(X, Y) + \eta(X)\eta(Y). \tag{7}$$

Then the structure (ϕ, ξ, η, g) satisfying conditions (2) to (7) is called an almost para-contact Riemannian structure and the manifold M^n with such a structure is called an almost para-contact Riemannian manifold [1, 19].

Now we briefly present an account of an analogue of the Kenmotsu manifold in paracontact geometry which will be called para-Kenmotsu.

Definition 1. The almost paracontact metric structure (ϕ, ξ, η, g) is para-Kenmotsu should this relation hold[2, 16], if the Levi-Civita connection ∇ of g satisfies $(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X$, for any $X, Y \in \mathfrak{X}(M)$.

On a para-Kenmotsu manifold [2, 20], the following relations hold:

$$\nabla_X \xi = X - \eta(X)\xi, \tag{8}$$

$$(\nabla_X \eta)Y = g(X, Y) - \eta(X)\eta(Y), \tag{9}$$

$$\eta(R(X, Y, Z)) = g(X, Z)\eta(Y) - g(Y, Z)\eta(X),$$
 (10)

$$R(X, Y, \xi) = \eta(X)Y - \eta(Y)X, \tag{11}$$

$$R(X, \xi, Y) = -R(\xi, X, Y) = g(X, Y)\xi - \eta(Y)X,$$
 (12)

$$S(\phi X, \phi Y) = -(n-1)g(\phi X, \phi Y), \tag{13}$$

$$S(X, \xi) = -(n-1)\eta(X),$$
 (14)
 $Q\xi = -(n-1)\xi.$ (15)

$$Q\xi = -(n-1)\xi,$$
 (15)
 $r = -n(n-1),$ (16)

for any vector fields X,Y,Z, where Q is the Ricci operator that is g(QX,Y)=S(X,Y), S is the Ricci tensor and r is the scalar curvature.

In A. M. Blaga [2], gave an example on para-Kenmotsu manifold:

Example 1. We consider the three dimensional manifold $M^3 = \{(x, y, z) \in \mathbb{R}^3, z \neq 0\}$, where (x, y, z) are the standard co-ordinates in \mathbb{R}^3 . The vector fields

$$e_1 := \frac{\partial}{\partial x}, e_2 := \frac{\partial}{\partial y}, e_3 := -\frac{\partial}{\partial z}$$

are linearly independent at each point of the manifold.

Define

 $\phi:=\frac{\partial}{\partial y}\otimes dx+\frac{\partial}{\partial x}\otimes dy, \xi:=-\frac{\partial}{\partial z}, \eta:=-dz.$

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$$g:=dx\otimes dx-dy\otimes dy+dz\otimes dz.$$

Then it follows that

$$\phi e_1 = e_2$$
, $\phi e_2 = e_1$, $\phi e_3 = 0$,
 $\eta(e_1) = 0$, $\eta(e_2) = 0$, $\eta(e_3) = 1$.

Let ∇ be the Levi-Civita connetion with respect to metric g. Then, we have

$$[e_1, e_2] = 0, [e_2, e_3] = 0, [e_3, e_1] = 0$$

The Riemannian connection ∇ of the metric g is deduced from Koszul's formula

$$\begin{split} 2g(\nabla_X Y, Z) &= X(g(Y, Z)) + Y(g(Z, X)) - Z(g(X, Y)) \\ &- g(X, [Y, Z]) + g(Y, [Z, X]) + g(Z, [X, Y]). \end{split}$$

Then Koszul's formula yields

$$\begin{split} &\nabla_{e_1}e_1 = -e_3, \nabla_{e_1}e_2 = 0, \nabla_{e_1}e_3 = e_1, \\ &\nabla_{e_2}e_1 = 0, \nabla_{e_2}e_2 = e_3, \nabla_{e_2}e_3 = e_2, \\ &\nabla_{e_3}e_1 = e_1, \nabla_{e_3}e_2 = e_2, \nabla_{e_3}e_3 = 0. \end{split}$$

These results shows that the manifold satisfies

$$\nabla_X \xi = X - \eta(X)\xi,$$

for $\xi=e_3$. Hence the manifold under consideration is para-Kenmotsu manifold of dimension three.

A para-Kenmotsu manifold is said to be an $\eta\textsc{-}\textsc{Einstein}$ manifold if its Ricci tensor S is of the form

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y) \tag{17}$$

for the vector fields X, Y, where a and b are functions on M^n .

3 Generalized pseudo-projective curvature tensor of para-Kenmotsu manifold

In this section, we give a brief account of generalized pseudo-projective curvature tensor of para-Kenmotsu manifold and study various geometric properties of it.

The pseudo-projective curvature tensor of para-Kenmotsu manifold ${\cal M}^n$ is given by the following relation:

$$\overline{P}(X,Y,U) = aR(X,Y,U) + b[S(Y,U)X - S(X,U)Y] - \frac{r}{n} \left(\frac{a}{n-1} + b\right) [g(Y,U)X - g(X,U)Y],$$

$$(18)$$

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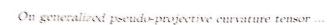


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Also, the type (0,4) tensor field \overline{P} is given by

$${}^{r}\overline{P}(X,Y,U,V) = a'R(X,Y,U,V) + b[S(Y,U)g(X,V) - S(X,U) g(Y,V)] - \frac{r}{n} \left(\frac{a}{n-1} + b\right) [g(Y,U)g(X,V) - g(X,U)g(Y,V)],$$
(19)

where

$${}^{\prime}\overline{P}(X,Y,U,V) = g(\overline{P}(X,Y,U),V)$$

and

$${}^{\prime}R(X,Y,U,V) = g(R(X,Y,U),V)$$

for the arbitrary vector fields X, Y, U, V.

Differentiating covariantly with respect to W in equation (18), we get

$$(\nabla_W \overline{P})(X, Y)U) = a(\nabla_W R)(X, Y)U) + b[(\nabla_W S)(Y, U)X - (\nabla_W S)(X, U)Y] - \frac{dr(W)}{n} \left(\frac{a}{n-1} + b\right) [g(Y, U)X - g(X, U)Y].$$
(20)

Divergence of pseudo-projective curvature tensor in equation (18) is given by

$$(div\overline{P})(X,Y)U) = a(divR)(X,Y)U) + b[(\nabla_X S)(Y,U) - (\nabla_Y S)(X,U)] - (divr) \left[\frac{a+b(n-1)}{n(n-1)}\right] [g(Y,U)div(X) - g(X,U)div(Y)].$$

$$(21)$$

But

$$(divR)(X,Y)U) = (\nabla_X S)(Y,U) - (\nabla_Y S)(X,U). \tag{22}$$

From equations (21) and (22), we have

$$(\operatorname{div}\overline{P})(X,Y)U) = (a+b)[(\nabla_X S)(Y,U) - (\nabla_Y S)(X,U)] - (\operatorname{div}r)$$

$$\left[\frac{a+b(n-1)}{n(n-1)}\right][g(Y,U)\operatorname{div}(X) - g(X,U)\operatorname{div}(Y)]. \tag{23}$$

Definition 2. An almost paracontact structure (ϕ, ξ, η, g) is said to be locally pseudo-projectively symmetric if

$$(\nabla_W \overline{P})(X, Y, U) = 0, \tag{24}$$

for all vector fields $X, Y, U, W \in T_pM^n$.

Definition 3. An almost paracontact structure (ϕ, ξ, η, g) is said to be locally pseudo-projectively ϕ -symmetric if

$$\phi^2((\nabla_W \overline{P})(X, Y, U)) = 0. \tag{25}$$

for all vector fields X, Y, U, W orthogonal to ξ .

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